Radiation effects on hydromagnetic free convective and mass transfer flow of a gas past a circular cylinder with uniform heat and mass flux

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M.A. EL-Hakiem
Department of Mathematics, Faculty of Science, South Valley University, Aswan, Egypt

Abstract

Purpose – The purpose of this paper is to highlight the effect of combined heat and mass transfer characteristics of magnetohydrodynamic (MHD) free convection flow of an electrically conducting Newtonian fluid on circular cylinder with uniform heat/mass flux, taking into consideration the effects of uniform transverse magnetic field and thermal radiation.

Design/methodology/approach – An analysis is performed to study the momentum, combined heat and mass transfer characteristics of MHD free convection flow past a circular cylinder surface under the effect of thermal radiation with uniform heat and mass flux. By using Lie group method, the infinitesimal generators of governing equations are calculated. Using the resulting generators for the boundary value problem, the equations are transformed into an ordinary differential system. Numerical solutions of the outcoming non-linear differential equations are found by using a combination of a Runge–Kutta algorithm and shooting technique.

Findings – Application of a magnetic field normal to the flow of an electrically conducting fluid gives rise to a resistive force that acts in the direction opposite to that of the flow. This resistive force tends to slow down the motion of the fluid along the cylinder and causes increases in its temperature and concentration and hence the respective changes in the wall shear stress, local Nusselt and Sherwood numbers as the magnetic parameter, respectively are changed with various values of angle which is measured in degrees from the front stagnation point on the surface. It is noted that these coefficients reduced as the magnetic parameter increases. Also, the effect of thermal radiation works as a heat source and so the quantity of heat added to the fluid increases, therefore the local Nusselt number reduced as the radiation parameter increases.

Research limitations/implications – An analysis is performed to study the momentum, combined heat and mass transfer characteristics of MHD free convection flow of an electrically conducting Newtonian fluid on circular cylinder with uniform heat/mass flux with the effects of uniform transverse magnetic field and thermal radiation.

Practical implications – This paper provides a very useful source of coefficient of heat and mass transfer values for engineers planning to transfer heat and mass by using electrically conducting gases with uniform heat/mass flux.

Originality/value – The combined heat and mass transfer of an electrically conducting gases on free convection flow in the presence of magneto and thermal radiation effects are investigated and can be used by different engineers working on industry, geothermal, geophysical, technological and engineering applications.

Keywords Magnetohydrodynamics, Flow, Heat transfer, Mass transfer

Paper type Research paper

1. Introduction

The radiative flow of an electrically conducting fluid and heat and mass transfer situation is of considerable interest because it can occur in many geothermal, geophysical, technological and engineering applications such as nuclear reactors,

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migration of moisture through air contained in fibrous insulations, grain storage, nuclear waste disposal, dispersion of chemical pollutants through water-saturated soil and others. The geothermal gases of constant wall temperature/concentration and constant heat/mass flux are electrically conducting and are affected by the presence of a magnetic field. Chamkha and Khaled (2000) have studied the effect of magnetic field on the coupled heat and mass transfer by mixed convection in a linearly stratified stagnation flow in the presence of an internal heat generation or absorption. EL-Hakiem (2000) studied thermal radiation effects on hydromagnetic free convection and flow through a highly porous medium bounded by a vertical plane surface. Borjini *et al.* (1999) have considered the effect of radiation on unsteady natural convection in a two-dimensional participating medium between two horizontal concentric and vertically eccentric cylinders. Chamkha (2000) has analyzed hydromagnetic mixed convection from a permeable semi-infinite vertical plate embedded in porous medium. Duwairi (2005) investigated radiation and magnetic field effects on forced convection flow from isothermal porous surfaces considering Viscous and Joule heating. Hayat (2006) reported the modeling and exact analytic solutions for hydromagnetic oscillatory rotating flows of an incompressible Burgers fluid bounded by a plate. The problem of magnetohydrodynamic (MHD) boundary layer flow of an upper-convected Maxwell fluid is investigated in a channel by Abbas *et al.* (2006). Khan *et al.* (2006) developed exact analytical solutions for the MHD flows of an Oldroyd-B fluid through a porous space. Hayat and Ali (2006) considered the MHD third grade fluid confined in a circular cylindrical tube. An analytic solution for the time-dependent flow of an incompressible third-grade fluid which is under the influence of a magnetic field of variable strength is investigated by Hayat and Kara (2006). Also, Hayat *et al.* (2006) proposed the equations for MHD flows of an Oldroyd-B fluid through a porous medium. Ayani *et al.* (2006) have showed the effect of radiation from the heat source and the variation of fluid properties on the laminar natural convection induced by a line heat source. Sajid *et al.* (2007) have investigated heat transfer characteristic in a third-order fluid over a linear stretching of a non-conducting sheet in the presence of a uniform applied magnetic field. Hayat *et al.* (2007b) presented the influence of thermal radiation on MHD flow of a second grade fluid. EL-Hakiem and Rashad (2007) proposed local non-similarity method to study the radiation effect on non-Darcy free convection flow over an isothermal vertical cylinder embedded in a saturated porous medium. Hayat *et al.* (2007a) studied the influences of heat transfer on an oscillatory flow of a non-Newtonian fluid through a porous medium.

On the other hand, it is now well known that the classical Lie symmetry method can be used to find similarity solutions, invariants, integrals motion, etc. systematically, see, for example, (Bluman and Kumei, 1989; Ovsiannikov, 1982; Olver, 1986; Ibragimov, 1999) and the usefulness of this approach has been widely illustrated by several authors in different contexts such as; Yurusoy and Pakdemirli (1997) found symmetry reductions of unsteady three-dimensional boundary layers of some non-Newtonian fluids. Also, Group classification of boundary layer equations of a non-Newtonial fluid flow problem has been performed by them (Yurusoy and Pakdemirli, (1999). Recently, Soh (2005) has used Lie symmetry techniques to obtain all non-similar and similarity reductions of a non-linear diffusion equation arising in the study of the flow of a charged non-Newtonian fluid over a flat plate. Soh *et al.* (2005) used symmetry methods to obtain non-equivalent similarity reductions of the steady two-dimensional thermal boundary layer equations of an incompressible laminar flow. Hayat *et al.* (2005) examined the unsteady flow of a hydrodynamic fluid past a porous plate by implementation of the Lie group method. Sivasankaran *et al.* (2006a, b) studied coupled heat and mass transfer fluid flow by natural convection past an inclined semi-infinite porous surface using Lie group analysis. The flow of a third-grade fluid occupying the space over a wall is studied analytically using Lie group methods by Hayat *et al.* (2003). Mohyuddin *et al.* (2004) applied Lie symmetry group method to obtain some steady as well as unsteady solutions of the equations of motion for an incompressible Newtonian and non-Newtonian fluids. Hayat and Mahomed (2007) obtained a new exact power law solution for the pipe flow of a third-grade fluid. Hayat and Kara (n.d.) presented here deals with similarity solutions of the problem of the flow of a third grade fluid past an infinite plate which are in a state of rigid body rotation. Hayat *et al.* (2007c) studied the flow generated in a semi-infinite expanse of an incompressible second-grade fluid bounded by a porous oscillating disk in the presence of a uniform transverse magnetic field. El-Kabeir *et al.* (2007) have applied Group method to simulate problem of heat and mass transfer in boundary-layer flow of an electrically conducting fluid over a vertical permeable cone surface saturated porous medium in the presence of a uniform transverse magnetic field and thermal radiation effects. EL-Kabeir *et al.* (2008) investigated Lie group method for solving the problem of heat transfer in an unsteady, three-dimensional, laminar, boundary layer flow of a viscous and electrically conducting fluid over inclined permeable surface embedded in porous medium in the presence of a uniform magnetic field. The analytical solutions for two thin film flow problems on a moving belt has been obtained using Lie point symmetry generators by Asghar *et al.* (2007).

This paper is concerned with the solution of the problem of combined heat and mass transfer characteristics of MHD natural convection flow past a circular cylinder surface under the effect of thermal radiation with uniform heat and mass flux. Lie group theory is applied to the equations of motion describing the flow invariant. The symmetries of the equations are found. The equations admit seven Lie point symmetries. To obtain a group invariant solution, we use scaling symmetries that is, scaling in x coordinate is used to transform the partial differential system into an ordinary differential system. Since the resulting differential equations are more involved, we have obtained reduction by one in the number of independent variable in the system. These reductions are continued until a system of ordinary differential equations is reached. Then similarity solution of these ordinary differential equations (ODEs) is found to discuss and plot both effects of magnetic field and radiation on velocity, temperature and concentration distributions as well as wall shear stress, local Nusselt and Sherwood numbers.

2. Mathematcal analysis

Let us consider the problem of heat and mass transfer characteristics in steady MHD natural convection flow past a vertical circular cylinder, where an orthogonal curvilinear co-ordinate system $(\bar{x}, \bar{y})$ in which $\bar{x}$ is measured along the surface of the cylinder from the front stagnation point and $\bar{y}$ normal to the surface of the cylinder. The applied magnetic field is in the y-direction. The magnetic Reynolds number is assumed to be small, which is true in most of the laboratory conducting fluids, so that the applied magnetic field is hardly affected by the induced magnetic field. Also, the radiative heat flux in the x-direction is considered negligible in comparison to that in the y-direction. Thus, the non-dimensional equations governing the steady flow of a Newtonian fluid past a circular cylinder this co-ordinate system are:

Continuity:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial}{\partial \bar{y}}[(1 + K_1\bar{y})\bar{v}] = 0, \tag{1}$$

Momentum:

$$\frac{\bar{u}}{(1+K_1\bar{y})}\left(\frac{\partial \bar{u}}{\partial \bar{x}} + K_1\bar{v}\right) + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{(1+K_1\bar{y})}\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{1}{\mathrm{Re}}\left[\frac{1}{(1+K_1\bar{y})^2}\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{2K_1}{(1+K_1\bar{y})^2}\frac{\partial \bar{v}}{\partial \bar{x}} - \frac{\bar{y}}{(1+K_1\bar{y})^3}\frac{\partial \bar{u}}{\partial \bar{x}}\frac{dK_1}{d\bar{x}} + \frac{K_1}{(1+K_1\bar{y})}\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\bar{v}}{(1+K_1\bar{y})^3}\frac{dK_1}{d\bar{x}} - \frac{K_1^2\bar{u}}{(1+K_1\bar{y})^2}\right] - \frac{\sigma \bar{B}_0^2}{\mu \mathrm{Re}}\bar{u} \tag{2}$$

$$\frac{\bar{u}}{(1+K_1\bar{y})}\left(\frac{\partial \bar{v}}{\partial \bar{x}} - K_1\bar{u}\right) + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = -\frac{\partial \bar{p}}{\partial \bar{y}} + \frac{1}{\mathrm{Re}}\left[\frac{1}{(1+K_1\bar{y})^2}\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} - \frac{2K_1}{(1+K_1\bar{y})^2}\frac{\partial \bar{u}}{\partial \bar{x}} - \frac{\bar{y}}{(1+K_1\bar{y})^3}\frac{\partial \bar{v}}{\partial \bar{x}}\frac{dK_1}{d\bar{x}} + \frac{K_1}{(1+K_1\bar{y})}\frac{\partial \bar{v}}{\partial \bar{y}} - \frac{\bar{u}}{(1+K_1\bar{y})^3}\frac{dK_1}{d\bar{x}} - \frac{K_1^2\bar{v}}{(1+K_1\bar{y})^2}\right] \tag{3}$$

Energy:

$$\frac{\bar{u}}{(1+K_1\bar{y})}\frac{\partial \bar{\theta}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{\theta}}{\partial \bar{y}} = \frac{1}{\mathrm{PrRe}}\left[\frac{1}{(1+K_1\bar{y})}\frac{\partial^2 \bar{\theta}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{\theta}}{\partial \bar{y}^2} - \frac{\bar{y}}{(1+K_1\bar{y})^3}\frac{\partial \bar{\theta}}{\partial \bar{x}}\frac{dK_1}{d\bar{x}} + \frac{K_1}{(1+K_1\bar{y})}\frac{\partial \bar{\theta}}{\partial \bar{y}}\right] - \frac{1}{\rho C_p \mathrm{Re}}\frac{\partial q^r}{\partial \bar{y}} \tag{4}$$

Concentration:

$$\frac{\bar{u}}{(1+K_1\bar{y})}\frac{\partial \bar{\phi}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{\phi}}{\partial \bar{y}} = \frac{1}{Sc.\mathrm{Re}}\left[\frac{1}{(1+K_1\bar{y})^2}\frac{\partial^2 \bar{\phi}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{\phi}}{\partial \bar{y}^2} - \frac{\bar{y}}{(1+K_1\bar{y})}\frac{\partial \bar{\phi}}{\partial \bar{x}}\frac{dK_1}{d\bar{x}} - \frac{K_1}{(1+K_1\bar{y})}\frac{\partial \bar{\phi}}{\partial \bar{y}}\right] \tag{5}$$

where K_1 is the surface curvature.

In the above equation $\bar{u}$ and $\bar{v}$ are the components of velocity along $\bar{x}$ and $\bar{y}$ directions, respectively, $\bar{p}$ is the pressure, σ is electrical conductivity, $\bar{B}_0$ is the magnetic induction, μ dynamic viscosity, ν is the kinematic viscosity, ρ is the density of fluid, $\bar{\theta}$ is temperature and $\bar{\phi}$ is dimensionless species concentration. We have non-dimensionalised the co-ordinates by the radius of the circular cylinder R, velocities by

U_∞ (oncoming free stream velocity) and the pressure by ρU_∞^2. The dimensionless temperature $\bar{\theta}$ is defined as $(T - T_\infty)/(q_w R/k)$ and the dimensionless concentration $\bar{w}$ is defined as $(C - C_\infty)/(m_w R/\rho D)$.

In addition, the radiative heat flux q^r is described according to the Rosseland approximation such that:

$$q^r = -\frac{4\sigma_1}{3\chi}\frac{\partial T^4}{\partial \bar{y}}, \tag{6}$$

where σ_1 and χ are the Stefan–Boltzmann constant and the mean absorption coefficient, respectively. As done by Raptis (1998), the fluid-phase temperature differences within the flow are assumed to be sufficiently small so that T^4 may be expressed as a linear function of temperature. This is done by expanding T^4 in a Taylor series about the free-stream temperature T_∞ and neglecting higher-order terms to yield

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \tag{7}$$

By using Equations (6) and (7) in the last term of Equation (3), one obtains

$$\frac{\partial q^r}{\partial \bar{y}} = -\frac{16\sigma_1 T_\infty^3}{3\chi}\frac{\partial^2 T}{\partial \bar{y}^2}. \tag{8}$$

The various dimensionless parameters entering into the equations are

$$\mathrm{Re} = \frac{\rho U_\infty R}{\mu}, \quad M = \frac{\sigma \bar{B}_0^2}{\mu \mathrm{Re}}, \quad \mathrm{Pr} = \frac{\mu C_p}{k}, \quad Sc = \frac{\mu}{\rho D}, \tag{9}$$

where μ is the viscosity coefficient, k is the coefficients of heat conduction and C_p is the specific heat of the fluid at a constant pressure and ρ is the mass density. Re is the Reynolds number, M is magentic field parameter and Pr is the Prandtl number. D is the binary diffusion coefficient and Sc is the Schmidt number.

We now proceed to carry out the usual boundary-layer approach, as

$$\bar{u} = u,\ \bar{v} = \Omega v,\ \bar{x} = x,\ \bar{y} = \Omega y,\ \bar{p} = p,\ \bar{\theta} = \theta,\ \bar{\phi} = \phi,\ \bar{B}_0 = \frac{1}{\Omega}B_0, \tag{10}$$

where $\Omega = 1/\sqrt{\mathrm{Re}}$. Substituting Equation (10) into Equations (1)-(5) and collecting the coefficients of order unity, the governing equations within boundary layer approximation as $\Omega \to 0$, may be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{11}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - Mu, \tag{12}$$

$$\frac{\partial p}{\partial y} = 0, \tag{13}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\left(1 + \frac{4R_d}{3}\right)\frac{\partial^2 \theta}{\partial y^2}, \tag{14}$$

$$u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2}. \tag{15}$$

The boundary conditions are given by

$$u(x,0) = v(x,0) = 0, \tag{16a}$$

$$\left(\frac{\partial \theta(x,y)}{\partial y}\right)_{y=0} = \left(\frac{\partial \phi(x,y)}{\partial y}\right)_{y=0} = -1, \tag{16b}$$

$$u(x,\infty) = U(x) - Mx,\ \theta(x,\infty) = \phi(x,\infty) = 0, \tag{16c}$$

Equation (12) in the free stream is

$$(U - Mx)\left(\frac{dU}{dx} - M\right) + M(U - Mx) = -\frac{\partial p}{\partial x}, \tag{17}$$

Using Equation (17) into Equation (12) we get:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \frac{\partial^2 u}{\partial y^2} - M\left(u + x\frac{dU}{dx}\right). \tag{18}$$

The inviscid flow velocity on the surface of the cylinder is given as $U(x) = 2\sin(x)$.

3. Lie group analysis

The symmetry groups of Equations (11)-(15) are calculated using classical Lie group approach (Bluman and Kumei, 1989). The one-parameter infinitesimal Lie group of transformations leaving Equations (11)-(15) invariant is defined as

$$\begin{aligned}
x^* &= x + \varepsilon\xi_1(x,y,\psi,\theta,\phi),\\
y^* &= y + \varepsilon\xi_2(x,y,u,v,\theta,\phi),\\
u^* &= u + \varepsilon\eta_1(x,y,u,v,\theta,\phi),\\
v^* &= u + \varepsilon\eta_2(x,y,u,v,\theta,\phi)\\
\theta^* &= \theta + \varepsilon\eta_3(x,y,u,v,\theta,\phi),\\
\phi^* &= \phi + \varepsilon\eta_4(x,y,u,v,\theta,\phi),
\end{aligned} \tag{19}$$

One can confirm that the solutions of system (11)-(15) are given as follows:

$$\begin{aligned}
\xi_1 &= C_1 x + C_2,\\
\xi_2 &= a(x) + C_3 y + C_4,\\
\eta_1 &= (C_1 - 2C_3)u,\\
\eta_2 &= a'(x)u - C_3 v + h_1(x),\\
\eta_3 &= 2C_3\theta + h_2(x,y),\\
\eta_4 &= 2C_3\phi + h_3(x,y),
\end{aligned} \tag{20}$$

Imposing the restrictions from boundaries and from the boundary conditions on the infinitesimals, we obtain the following form for Equations (20)

$$\begin{aligned}
\xi_1 &= C_1 x + C_2,\\
\xi_2 &= C_3 y + C_4,\\
\eta_1 &= (C_1 - 2C_3)u,\\
\eta_2 &= -C_3 v,\\
\eta_3 &= 2C_3\theta,\\
\eta_4 &= 2C_3\phi,
\end{aligned} \tag{21}$$

where the parameters C_1 and C_2 represent the scaling transformations and parameter C_3 represents translation in the x coordinate. In the following sections, solutions corresponding to the above symmetries are derived.

4. Reduction to ordinary differential equations

In this section, translation in x coordinate is considered and hence we take $C_1 = C_2 = 0$. The characteristic equations for finding the similarity transformations would then be

$$\frac{\mathrm{d}x}{x} = \frac{\mathrm{d}y}{0} = \frac{\mathrm{d}u}{u} = \frac{\mathrm{d}v}{0} = \frac{\mathrm{d}\theta}{0} = \frac{\mathrm{d}\phi}{0}, \tag{22}$$

The similarity variables and resulting functions are

$$\lambda = y,\ u = xF_1'(\lambda),\ v = -F_1(\lambda),\ \theta = F_2(\lambda),\ \phi = F_3(\lambda), \tag{23}$$

where λ is the similarity variable and F_1, F_2 and F_3 are functions are to be determined. Finally, we can constrain the reduced system of ODEs corresponding to the above problem from Equations (11)-(15), respectively, by the functions F_1, F_2 and F_3 as follows:

$$F_1''' + F_1 F_1'' - F_1'^2 + \frac{\sin(2x)}{x} - M\big(F_1' + 2\cos(x)\big) = 0, \tag{24}$$

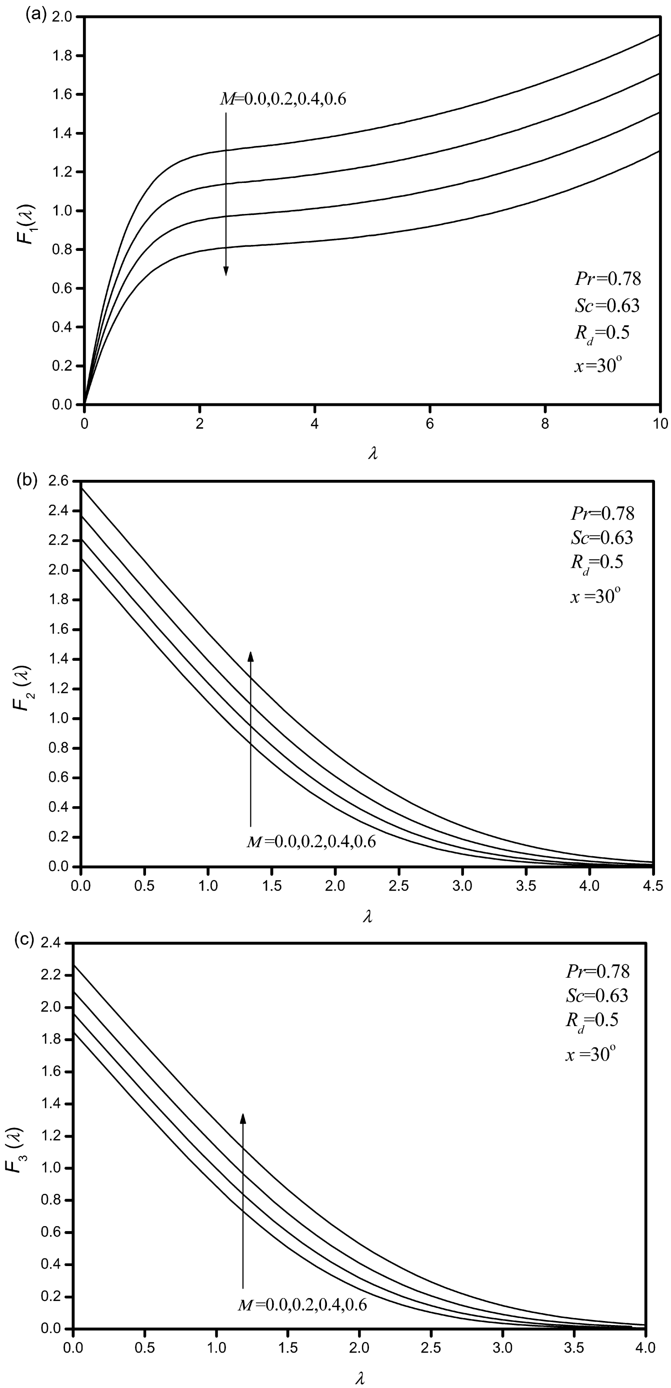

Figure 1.
Effect of magnetic parameter M on the distributions of (a) velocity, (b) temperature and (c) concentration

$$\left(1+\frac{4R_d}{3}\right)F_2'' + \mathrm{Pr}F_1F_2' = 0, \tag{25}$$

$$F_3'' + ScF_1F_3' = 0, \tag{26}$$

where the primes stand for differentiation with respect to the similarity variable λ. The associated boundary conditions (16) take the form:

$$F_1(0) = F_1'(0) = 0,\; F_2'(0) = F_3'(0) = -1, \tag{27a}$$

$$F_1'(\infty) = (2\sin(x))/x - M,\; F_2(\infty) = F_3(\infty) = 0, \tag{27b}$$

The physical quantities of interest are the skin friction on the circular cylinder, is defined as:

$$\tau_w = \left(\frac{\partial u}{\partial y}\right)_{y=0} = xF_1''(0), \tag{28}$$

the local Nusselt number, is defined as:

$$Nu_x = \frac{q_w R}{k(T_w - T_\infty)} = \frac{1}{F_2(0)}, \tag{29}$$

and Local Sherwood number, is defined as

$$Sh_x = \frac{m_w R}{\rho D(C_w - C_\infty)} = \frac{1}{F_3(0)}. \tag{30}$$

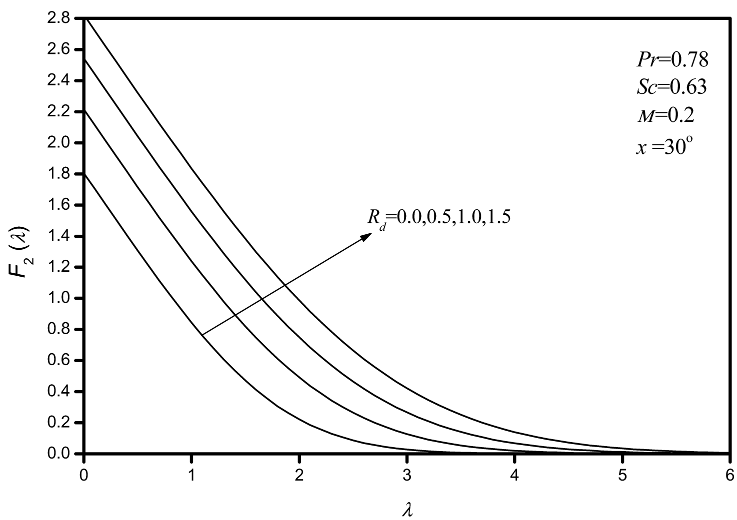

Figure 2. Effect of radiation parameter R_d on the distribution of temperature

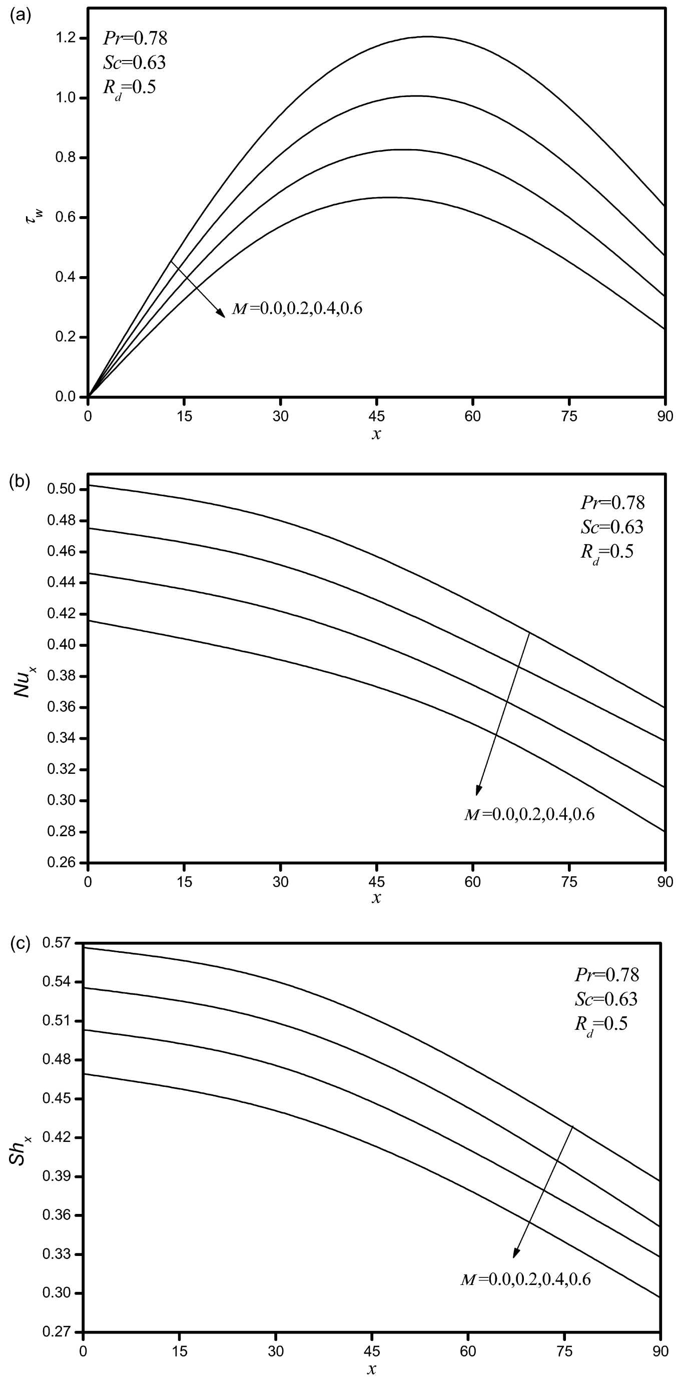

Figure 3.
Effect of magnetic parameter M on (a) wall shear stress, (b) local Nusselt number and (c) local Sherwood number

5. Results and discussions

The reduced Equations (24)-(26) together with boundary conditions (27), which represents the velocity, temperature and concentration profiles have been numerically solved using the fourth-order, Runge–Kutta numerical integration procedure in conjunction with shooting techniques. Numerical computation were carried numerically for Prandtl number Pr = 0.78 (metal ammonia), Schmidt number $Sc = 0.63$ (vapor water) and various values of magnetic field parameter M and radiation parameter R_d are recorded on the figures.

Figures 1(a)-(c) show the effect of dimensionless parameter of magnetic field M on the velocity, temperature and concentration distributions. Application of a magnetic field normal to the flow of an electrically conducting fluid gives rise to a resistive force that acts in the direction opposite to that of the flow. This force is called the Lorentz force. This resistive force tends to slow down the motion of the fluid along the cylinder and causes increases in its temperature and concentration. On other hand the effect of temperature distribution with various values of radiation parameter R_d is employed in Figure 2. Obviously, the distribution of temperature increases as the radiation parameter R_d increases. This result was expected because the presence of thermal radiation works as a heat source and so the quantity of heat added to the fluid increases. Also, this figure shows that the thermal boundary layer becomes thicker as R_d increases. Moreover, it is obvious that the governing Equations (24)-(26) are uncoupled. Therefore, changes in the values of R_d will cause no changes in both of the distributions of velocity and concentration of fluid, and for this reason, no figures for these variables are presented herein. Figures 3 and 4 depict the respective changes in the wall shear stress, local Nusselt number and Sherwood number as the magnetic and radiation parameters, respectively are changed with various values of angle x which is measured in degrees from the front stagnation point. It is noted that the wall shear stress, Nusselt number and Sherwood number reduced as the magnetic parameter M increases. Also, the local Nusselt number reduced as the radiation parameter R_d increases, but no effect on both of the wall shear stress and Sherwood number for the same reason as obvious above.

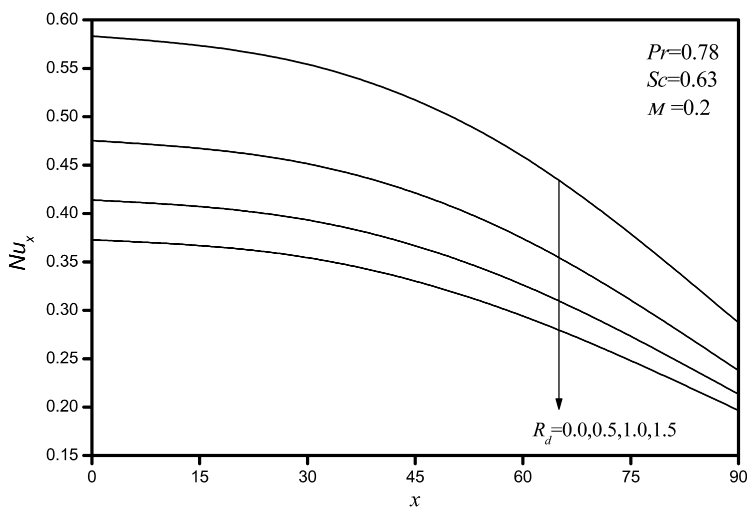

Figure 4.
Effect of radiation parameter R_d on local Nusselt number

6. Concluding remarks

Steady laminar boundary layer analysis of heat and mass transfer characteristics in a MHD fluid flow around a circular cylinder surface with uniform heat and mass flux boundary conditions were investigated. By employing Lie group analysis, the symmetries of the equations of motion are calculated. The Lie algebra consist of seven subgroups Lie group transformations, one being the scaling symmetry and the others being translations. Finally, an example set of boundary conditions for the solutions are considered to discuss the physical properties of the problem as well as local heat transfer rate and the local mass transfer rate with various values material parameters.

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Corresponding author
M.A. EL-Hakiem can be contacted at: hk_elhakiem@yahoo.com